

Heiser numbers

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1 Introduction

Willem, as my master thesis supervisor, was the one who introduced me to quantitative research. In my thesis I compared optimal scaling and item response theory. With his enthusiasm for quantitative measurement Willem made me excited about doing research in general, and about statistics in particular. Some years later I became interested in number theory, and I wrote a bachelor thesis on irrational and transcendental numbers. Since I am not particularly good with words, this paper is merely about numbers, more precisely, Heiser numbers. Compared to the definition of a transcendental number (which is a number that is not the root of a non-zero polynomial with rational coefficients), the definition of a Heiser number is pretty straightforward. The idea of a Heiser number is based on that of an Erdős number.

Paul Erdős was a mathematician from Hungary who studied and pursued problems in a number of mathematical fields, including, number theory, set theory, analysis, probability theory, combinatorics and graph theory. In the history of mathematics Erdős is considered one of the most prolific publishers of articles. He authored more than 1500 mathematical papers and collaborated with more than 500 different mathematicians. Most of his papers were written in collaboration with other mathematicians. Because of his prolific output, mathematicians many years ago came up with the Erdős number. An Erdős number describes a researcher's distance of collaboration from Erdős himself. Immediate collaborators of Erdős, researchers that have written papers with Erdős, have an Erdős number of 1. People that have collaborated with co-authors of Erdős have Erdős number at most 2, and so on. Erdős numbers are now a part of the folklore of mathematicians.

Computing the Erdős number of a researcher can be a formidable task, especially when a researcher has published with many different co-authors, or if a researcher comes from a field of mathematics that is somewhat remote from the research topics of Erdős. Willem has an Erdős number of at most 4. The shortest collaboration path I could find, with year of joint work in parentheses, is as follows: Erdős to Goddard (1994) to Cowen (1997) to Mather (1997) to Heiser (1995). The corresponding papers are:

Aronov, B., Erdős, P., Goddard, W., Kleitman, D.J., Klugerman, M., Pach, J. and Schulman, L.J. (1994). Crossing families. *Combinatorica*, 14, 127-134.

Cowen, L., Goddard, W. and Jesurum, C.E. (1997). Defective coloring revisited. *Journal of Graph Theory*, 24, 205-219.

Cowen, L. and Mathar, R. (1997). The offset problem. *Combinatorics, Probability and Computing*, 6, 159-164.

Groenen, P.J.F., Mathar, R. and Heiser, W.J. (1995). The majorization approach to multidimensional scaling for Minkowski distances. *Journal of Classification*, 12, 3-19.

It may well be that a shorter path between Willem and Erdős exists. We therefore say that Willem's Erdős number is at most 4.

2 Heiser numbers

In this chapter I consider the collaboration distances between Willem and various quantitative researchers. The collaboration distance between Willem and an author will be called the Heiser number of the author. Immediate collaborators of Willem have a Heiser number of 1. There are 73 researchers with a Heiser number of 1. Willem has collaborated with several people that, in turn, have collaborated with many other authors. Examples are Spinhoven, Sokal and Mathar. The number of people with a Heiser number of 2 must therefore run in the hundreds. Moreover, the number of people with a Heiser number of at most 3 will probably run in the thousands. I have therefore not attempted to produce a complete list of people with a Heiser number of at most 2 or 3.

Some examples of people with a Heiser number of 1 and 2 are presented in Table 1. The first column of Table 1 presents authors that have a Heiser number of 2. The second column presents authors that have a Heiser number of 1. For the latter authors the year of joint work with Willem is in parentheses behind their names. The authors in the second column connect the researchers in the first column to Willem. The third column contains the year that the authors in the first and second columns collaborated.

Author with a Heiser number of 2	Author with a Heiser number of 1	Year of joint publication of the two authors
Friedman, J.H.	Meulman (2000)	2004
Tukey, J.W.	Meyer (1987)	1999
Fienberg, S.E.		1983
Sneath, P.H.A.	Sokal (1987)	1973
Rohlf, F.J.		1966
Mirkin, B.	Van Mechelen (2007)	2008
Gelman, A.		2005
Bock, H.-H.		2004
Rubin, D.B.	Van Buuren (1989)	2006
Gower, J.C.	De Rooij (2005)	2003
Anderson, C.J.		2007
Fichet, B.	Critchley (1988)	1997
DeSarbo, W.	De Soete (1993)	1986
Steinley, D.	Hubert (2000)	2008
Greenacre, M.	Kroonenberg (1988)	2006
Joly, S.	Bennani Dosse (1997)	2007

Table 1: Examples of reseachers with a Heiser number of 1 and 2.

In the remaining sections of this paper the Heiser numbers of two groups of people are presented. The first group consists of the past, present and future presidents of the Psychometric Society. The second group consists of selected past and present staff members of IOPS (Interuniversity Graduate School of Psychometrics and Sociometrics). The second group is a clear example of a small-world network. These networks are characterized by small average collaboration paths and many tightly knit groups of researchers.

3 Psychometric Society presidents

The Psychometric Society is an international professional organization for researchers in quantitative measurement. Table 2 presents the names and year of presidency of 58 past, present and future presidents of the Psychometric Society. Willem was president of the Psychometric Society in 2003-04. For 21 presidents no collaboration path to Willem was found. These presidents include Samejima, Fischer, Guttman and Winer. However, Thurstone, the society's first president in 1935-36, has a Heiser number of at most 4.

2014-15	Rabe-Hesketh, S.	2	1982-83	Bentler, P.	2
2013-14	Maydeu-Olivares, A.	4	1981-82	Ramsay, J.O.	3
2012-13	Chang, H.H.	4	1980-81	Young, F.W.	2
2011-12	Wilson, M.	2	1979-80	Novick, M.R.	4
2010-11	Sijtsma, K.	2	1978-79	Cliff, N.	3
2009-10	Ten Berge, J.	2	1977-78	Jöreskog, K.G.	5
2008-09	Junker, B.	3	1976-77	Luce, R.D.	3
2007-08	De Boeck, P.A.L.	1	1975-76	Carroll, J.D.	2
2006-07	Millsap, R.S.	6	1974-75	Kruskal, J.B.	3
2005-06	Böckenholt, U.	3	1973-74	Shepard, R.N.	3
2004-05	Cudeck, R.	5	1972-73	Bock, R.D.	3
2003-04	Heiser, W.J.	0	1971-72	Messick, S.J.	5
2002-03	Meulman, J.J.	1	1969-70	Kaiser, H.F.	5
2001-02	Stout, W.	4	1966-67	Harris, C.W.	6
2000-01	Thissen, D.	3	1965-66	Green, B.F., Jr.	3
1999-00	Van der Linden, W.J.	3	1964-65	Torgerson, W.S.	3
1998-99	Embretson, S.E.	2	1963-64	Edwards, A.L.	5
1997-98	Molenaar, I.W.	2	1962-63	Jones, L.V.	3
1995-96	Nishisato, S.	4	1959-60	Humphreys, L.G.	4
1993-94	Mislevy, R.J.	2	1958-59	Lord, F.M.	5
1992-93	Meredith, W.	5	1957-58	Mosteller, F.	3
1991-92	Browne, M.W.	4	1955-56	Coombs, C.H.	6
1990-91	Arabie, P.	2	1954-55	Tucker, L.R.	3
1989-90	Holland, P.W.	3	1953-54	Cronbach, L.J.	6
1988-89	Muthen, B.O.	4	1945-46	Cureton, E.E.	3
1987-88	De Leeuw, J.	1	1944-45	Gulliksen, H.	3
1986-87	Takane, Y.	2	1941-43	Horst, P.	6
1985-86	McDonald, R.P.	3	1940-41	Dunlap, J.W.	4
1983-84	Hubert, L.J.	1	1935-36	Thurstone, L.L.	4

Table 2: Heiser numbers of 58 presidents of the Psychometric Society.

The single peaked distribution of the 58 Heiser numbers of the presidents is as follows:

- Heiser number 0 --- 1 person
- Heiser number 1 --- 4 people
- Heiser number 2 --- 12 people
- Heiser number 3 --- 19 people
- Heiser number 4 --- 10 people
- Heiser number 5 --- 7 people
- Heiser number 6 --- 5 people

The median Heiser number of the presidents is 3; the mean is 3.3 and the standard deviation is 1.9. Four presidents have collaborated with Willem directly, resulting in a Heiser number of 1: De Leeuw (1981), Meulman (1999), Hubert (2000) and De Boeck (2013).

There are 12 presidents that have a Heiser number of 2. Ten Berge (1987), Bentler (2011), Young (1977) and Takane (1977) have collaborated with De Leeuw, while Molenaar (1988) and Mislavy (1990) have published with Verhelst (2009). Furthermore, Wilson (2004), Embretson (in press) and Rabe-Hesketh (in press) have collaborated with De Boeck (2013). In addition, we have the paths, Heiser to Bouwmeester (2003) to Sijtsma (2007), Heiser to Hubert (2000) to Arabie (1985), and Heiser to De Soete (1981) to Carroll (1983).

There are 19 presidents that have a Heiser number of at most 3. Several presidents can be connected to Willem via the famous mathematician Tukey, the developer of the box plot. It turns out that Tukey has a Heiser number of at most 2. Tukey and Willem are connected by Meyer. One collaboration path consists of the following two papers:

Kadane, J.B., Meyer, M.M. and Tukey, J.W. (1999). Yule's association paradox and ignored stratum heterogeneity in capture-recapture studies. *Journal of the American Statistical Association*, 94, 855-859.

Gittins, R., Amir, S., Dupouey, J.-L., Heiser, W.J., Meyer, M.M., Sokal, R.R. and Werger, M.J.A. (1987). Numerical methods in terrestrial plant ecology. In P. Legendre et al. (Eds.) *Developments in Numerical Ecology*. Springer, New York, pp. 529-558.

Co-authors of Tukey are Luce (1964), Mosteller (1953), Green (1960), Jones (1999), Cureton (1951) and Gulliksen (1958). These 6 presidents therefore have a Heiser number of at most 3.

Other presidents with a Heiser number of at most 3 are, McDonald due to a collaboration with Steinley (2007), Shepard due to a collaboration with Arabie (1979), Junker due to a collaboration with Sijtsma (1996), Böckenholt due to a collaboration with Anderson (2000), Kruskal due to a collaboration with Carroll (1980), and Bock due to a collaboration with Mislavy (1982). Furthermore, both Torgerson (1967) and Cliff (1968) have published with Young. We also have the following paths: Heiser to De Soete (1993) to Winsberg (2002) to Ramsay (1980), Heiser to Verhelst (2009) to Glas (1993) to Van der Linden (2003), Heiser to Hubert (2000) to Wainer (2011) to Thissen (1982), Heiser to Meyer (1987) to Fienberg (1983) to Holland (1972), and Heiser to Kroonenberg (1998) to Lewis (1982) to Tucker (1973).

There are 10 presidents that have a Heiser number of at most 4. Among them is Thurstone, the first president of the Psychometric Society, due to a publication with Jones (1957). Other presidents with a Heiser number of at most 4 are, Nishisato due to a collaboration with McDonald (1979), Novick due to a collaboration with Ramsay (1980), Maydeu-Olivares due to a collaboration with Böckenholt (2005), Dunlap due to a collaboration with Cureton (1930), and Humphreys due to a collaboration with Bock and Green (1984). In addition, we have the following paths: Heiser to Meyer (1987) to Tukey (1999) to Bickel (1972) to Chang (2001), Heiser to Mathar (1995) to Pfeifer (1990) to Serfling (1988) to Stout (1982), Heiser to De Leeuw (1981) to Ten Berge (1987) to Shapiro (1999) to Browne (1987), and Heiser to De Leeuw (1981) to Bentler (2011) to Satorra (1990) to Muthen (1995).

There are 7 presidents with a Heiser number of at most 5. The first is Lord, who has worked with Novick (1968). Another shortest path between Willem and Lord goes like this: Heiser to De Rooij (2000) to Gower (2003) to Laslett (1978) to Cressie (1987) to Lord (1975). Other presidents with a Heiser number of at most 5 are, Cudeck due to a collaboration with Browne (1989), Edwards due to a collaboration with Thurstone (1952), and Meredith (1972) and Messick (1957) due to a collaboration with Tucker. We also have the following paths:

Heiser to Groenen (1995) to Zha (2006) to Golub (1994) to Ortega (1991) to Kaiser (1963/1964), and Heiser to Groenen (1995) to Lesaffre (2013) to Tsonaka (2006) to Moustaki (2007) to Jöreskog (2004).

Finally, there are 5 presidents with a Heiser number of at most 6. A shortest path from Heiser to Coombs is as follows: Heiser to Meyer (1987) to Fienberg (1983) to Holland (1972) to Laskey (1983) to Lehner (1989) to Coombs (1984). Other presidents with a Heiser number of at most 6 are, Millsap due to a collaboration with Meredith (1975), Harris due to a collaboration with Kaiser (1964), and Cronbach (1952) and Horst (1953) due to a collaboration with Edwards. Note that the paths from Willem to Millsap, Willem to Cronbach, and Willem to Horst, each involve 5 presidents of the Psychometric Society: Heiser, Gulliksen, Tucker, Meredith, Millsap; Heiser, Jones, Thurstone, Edwards, Cronbach or Horst.

4 IOPS staff members

Quantitative researchers from the social and behavioral sciences in the Netherlands are organized in the Interuniversity Graduate School of Psychometrics and Sociometrics. In Dutch the title of the institute is abbreviated as IOPS. The institute coordinates the research in psychometrics and sociometrics and provides dissertation training. Willem has been director of IOPS from 2006 till 2013. Table 3 presents the Heiser numbers of 108 selected past and present IOPS staff members. For some staff members no collaboration path to Willem could be found. The single-peaked distribution of the 108 IOPS staff members is as follows:

Heiser number 1 --- 15 people
Heiser number 2 --- 30 people
Heiser number 3 --- 51 people
Heiser number 4 --- 7 people
Heiser number 5 --- 4 people
Heiser number 6 --- 1 person

Thus the median Heiser number of the selected IOPS members is 3; the mean is 2.61, and the standard deviation is 1. Half of the people that have a Heiser number of 1 are former PhD students of Willem: De Rooij (2005), Frank (2008), Groenen (1996), Hickendorff (2009), Polak (2012), Van Buuren (1989), and Warrens (2009). Other researchers with a Heiser number of 1 are, Bouwmeester (2003), De Boeck (2013), Kroonenberg (1998), Meulman (2000), Van Mechelen (2007), Van Putten (2009), van der Kloot (2005), and Verhelst (2009).

There are 30 IOPS staff members that have a Heiser number of at most 2. Bechger (2002), Dolan (2010), Eggen (2011), Glas (1989), Jansen (1997), Maris (2004), Molenaar (1988), and Van Schuur (1980) have collaborated with Verhelst, whereas Kiers (2011), Ten Berge (2008) and Stegeman (2009) have collaborated with Bennani Dosse. Furthermore, Arends (2012), Emons (2008), Sijtsma (2007) and Vermunt (2007) have collaborated with Bouwmeester, and Oort (2003) and Van Ginkel (2012) have collaborated with Kroonenberg. Moreover, Van der Heijden (1985) and Mooijaart (1989) have collaborated with De Leeuw, Braeken (2007) and Grasmann (2011) have collaborated with De Boeck, Eilers (2013) and Van der Ark (2006) have collaborated with Groenen, Hoijsink (2009) and Ceulemans (2013) have collaborated with Van Deun (2007), and Tuerlinckx (2000) and Schepers (2006) have collaborated with Van Mechelen. Other researchers with a Heiser number of at most 2 are, Van der Leeden (1999) due to a collaboration with Busing (2005), Albers (2011) due to a collaboration with Critchley, and Smit (2009) due to a collaboration with Spinhoven (2009). Due to collaborations with other IOPS staff members, almost half of the selected IOPS staff members

in Table 3 have a Heiser number of at most 3. With a small average collaboration path and many tightly knit groups of researchers, IOPS is an example of a small-world network.

Albers, C.J.	2	Huisman, J.M.E.	3	Tan, F.E.S.	4
Arends, L.R.	2	Huizinga, H.M.	3	Tekle, F.B.	3
Bechger, T.M.	2	Jansen, B.R.J.	3	Ten Berge, J.M.F.	2
Béguin, A.A.	3	Jansen, M.G.H.	2	Timmerman, M.E.	3
Berger, M.P.F.	4	Kankaraš, M.	3	Tuerlinckx, F.	2
Bethlehem, J.G.	5	Kelderman, H.	3	Van Assen, M.A.L.M.	3
Boomsma, A.	3	Kiers, H.A.L.	2	Van Breukelen, G.J.P.	4
Borsboom, D.	3	Klugkist, I.G.	3	Van Buuren, S.	1
Bouwmeester, S.	1	Kroonenberg, P.M.	1	Van Duijn, M.A.J.	3
Braeken, J.	2	Kuiper, R.M.	3	Van Ginkel, J.R.	2
Candel, M.J.J.M.	5	Lensvelt-Mul., G.	3	Van Mechelen, I.	1
Ceulemans, E.	2	Ligtvoet, R.	3	Van Putten, C.M.	1
Croon, M.A.	3	Maris, G.K.J.	2	Van Schuur, W.H.	2
Cruyff, M.J.L.F.	3	Meijer, R.R.	3	Vanpaemel, W.	3
De Boeck, P.A.L.	1	Mellenbergh, G.J.	3	Vd Ark, L.A.	2
De Leeuw, E.D.	4	Meulman, J.J.	1	Vd Berg, S.M.	3
De Rooij, M.	1	Moerbeek, M.	3	Vd Heijden, P.G.M.	2
Dijkstra, W.	5	Mokken, R.J.	3	Vd Kloot, W.A.	1
Dolan, C.V.	2	Molenaar, I.W.	2	Vd Leeden, R.	2
Eggen, T.J.H.M.	2	Mooijaart, A.	2	Vd Linden, W.J.	3
Eilers, P.H.C.	2	Moors, G.B.D.	3	Vd Maas, H.L.J.	3
Emons, W.H.M.	2	Morey, R.D.	5	Vd Schoot, A.G.J.	3
Evers, A.V.A.M.	3	Mulder, J.	3	Vd Sluis, S.	3
Fox, G.J.A.	3	Oberski, D.	3	Veldkamp, B.P.	3
Frank, L.E.	1	Ongena, Y.P.	6	Verhagen, A.J.	4
Gelissen, J.P.T.M.	3	Oort, F.J.	2	Verhelst, N.	1
Glas, C.A.W.	2	Polak, M.G.	1	Vermunt, J.K.	2
Grasman, R.P.P.P.	2	Raijmakers, M.E.J.	3	Visser, I.	3
Groenen, P.J.F.	1	Saris, W.E.	4	Vos, H.J.	3
Hagenaars, J.A.P.	3	Schepers, J.	2	Wagenmakers, E.M.	3
Hamaker, E.L.	3	Schmittmann, V.D.	3	Waldorp, L.J.	3
Hemker, B.T.	3	Sijtsma, K.	2	Warrens, M.J.	1
Hessen, D.J.	3	Smits, N.	2	Wicherts, J.M.	3
Hickendorff, M.	1	Snijders, T.A.B.	3	Zand Scholten, A.	3
Hoijtink, H.J.A.	2	Snijkers, G.	4	Zijlstra, B.J.H.	3
Hox, J.J.	3	Stegeman, A.W.	2	Zijlstra, W.P.	3

Table 3: Heiser numbers of 108 selected past and present IOPS staff members.